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CREEP OF PLATES MADE OF ALUMINUM ALLOYS UNDER BENDING

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Results of experiments dealing with the creep of a V95pchT2 *aluminum alloy are presented. Constitutive equations of steady creep are constructed and used to solve the problem of pure twisting of a square plate. Calculated and experimental values of plate curvature are compared.*

Key words: *aluminum alloys, creep, different resistance to tension and compression, twisting of a square plate.*

Advanced structural alloys possess some specific features in terms of their strains and strength, such as different resistance to tension and compression and anisotropy. These features are most vividly manifested at elevated temperatures [1].

1. Experimental Data. Constitutive Equations. To determine the characteristics of creep of the V95pchT2 aluminum alloy under tension and compression at a temperature $T = 180°$C, samples were prepared from plates 40 and 50 mm thick; the samples were cut in different directions (longitudinal, transverse, and normal to the plate). The experiments were performed at different stresses σ constant in time (270 MPa $\le |\sigma| \le$ 320 MPa). The experimental data for steady-state creep were processed by using a power dependence between the strain rate η and stress σ:

$$\sigma > 0: \quad \eta = B_1\sigma^{n_1}, \qquad \sigma < 0: \quad \eta = B_2|\sigma|^{n_2-1}\sigma. \tag{1}$$

The following values of constants were obtained:

— for the plate 50 mm thick, $B_1 = 7.2 \cdot 10^{-31}$ MPa$^{-n_1}\cdot$sec^{-1} and $B_2 = 2 \cdot 10^{-31}$ MPa$^{-n_2}\cdot$sec^{-1} ($n_1 = n_2 = 10$);

— for the plate 40 mm thick, $B_1 = 6 \cdot 10^{-55}$ MPa$^{-n_1}\cdot$sec^{-1} ($n_1 = 20$) and $B_2 = 2.43 \cdot 10^{-43}$ MPa$^{-n_2}\cdot$sec^{-1} ($n_2 = 15$).

Thus, the creep properties of the same alloy substantially depend on plate thickness. The experimental results allow us to believe (at least, in the first approximation) that the V95pchT2 material at $T = 180°$C is isotropic but displays different properties under tension and compression. Them, similar to [2], dependence (1) can be extended to the complex stress state:

$$\begin{gathered}\eta_{kl} = \frac{\partial\Phi}{\partial\sigma_{kl}}, \qquad 2\Phi(\sigma_i, \xi) = \Phi_1 + \Phi_2 + (\Phi_2 - \Phi_1)\sin 3\xi, \\ \Phi_1 = B_1\sigma_i^{n_1+1}/(n_1+1), \qquad \Phi_2 = B_2\sigma_i^{n_2+1}/(n_2+1).\end{gathered} \tag{2}$$

Here η_{kl} and σ_{kl} ($k, l = 1, 2, 3$) are the components of the creep strain rate and stress tensors in the chosen coordinate system $Ox_1x_2x_3$, Φ is the creep potential, σ_i is the stress intensity, and ξ is the angle of the form of the stress state.

The quantities σ_i and ξ were defined in [2]. In a plane stress state, like in the case of plate bending considered below, we have

$$\sigma_i^2 = \sigma_{11}^2 + \sigma_{22}^2 - \sigma_{11}\sigma_{22} + 3\sigma_{12}^2, \qquad \sin 3\xi = (1/2)(I/\sigma_i)^3 - (3/2)(I/\sigma_i), \tag{3}$$

where $I = \sigma_{11} + \sigma_{22}$.

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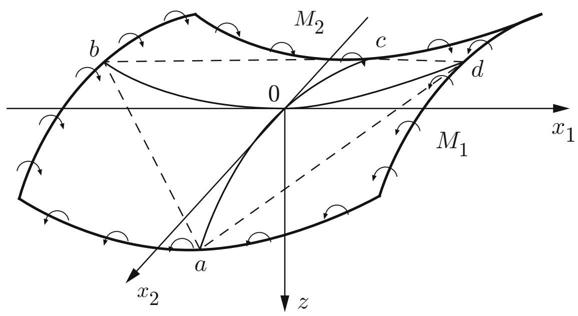

Fig. 1

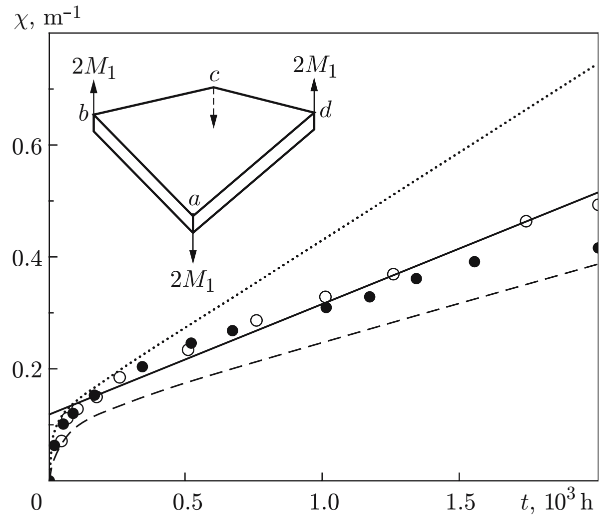

Fig. 2

Fig. 1. Loading of the plate by the torque M.

Fig. 2. Experimental (points) and theoretical (curves) dependences of plate curvature on time: the solid curve refers to the calculation for a kinematic regime of shape forming ($A_1 = 0.95$ m^{-1} and $A_2 = 0.5$ m^{-1}); the dotted curve refers to the calculation based on material characteristics under tension ($n_1 = n_2 = 20$ and $B_1 = B_2 = 6 \cdot 10^{-55}$ MPa$^{-20} \cdot$ sec^{-1}); the dashed curve refers to the calculation based on material characteristics under compression ($n_1 = n_2 = 15$ and $B_1 = B_2 = 2.43 \cdot 10^{-43}$ MPa$^{-15} \cdot$ sec^{-1}).

From Eqs. (2) and (3), we obtain

$$2\Phi(\sigma_{11}, \sigma_{22}, \sigma_{12}) = \Phi_1(\sigma_i) + \Phi_2(\sigma_i) + [\Phi_2(\sigma_i) - \Phi_1(\sigma_i)](\zeta^3 - 3\zeta)/2, \qquad \zeta = I/\sigma_i,$$

$$\eta_{11} = \Phi_3(2\sigma_{11} - \sigma_{22}) + \Phi_4(\sigma_{22}^2 - \sigma_{11}\sigma_{22} + 2\sigma_{12}^2),$$

$$\eta_{22} = \Phi_3(2\sigma_{22} - \sigma_{11}) + \Phi_4(\sigma_{11}^2 - \sigma_{11}\sigma_{22} + 2\sigma_{12}^2),$$

$$\eta_{12} = 3\Phi_3\sigma_{12} - \Phi_4(\sigma_{11} + \sigma_{22})\sigma_{12}, \tag{4}$$

$$\Phi_3 = (B_1\sigma_i^{n_1-1} + B_2\sigma_i^{n_2-1})/4 + (B_2\sigma_i^{n_2-1} - B_1\sigma_i^{n_1-1})(\zeta^3 - 3\zeta)/8,$$

$$\Phi_4 = \frac{9}{8}\Big(\frac{B_2}{n_2+1}\,\sigma_i^{n_2-2} - \frac{B_1}{n_1+1}\,\sigma_i^{n_1-2}\Big)(\zeta^2 - 1).$$

Note that, for $n_1 \neq n_2$, the potential Φ from Eqs. (2), in contrast to the commonly used potentials, is not a uniform function and depends on the second and third invariants of the stress deviator.

In determining the instantaneous elastic characteristics of the examined material, the V95pchT2 alloy at $T = 180$°C was found to be isotropic and to have identical properties under tension and compression: Young's modulus $E = 57$ GPa and Poisson's ratio $\nu = 0.4$.

2. Twisting of a Square Plate. Let us consider a square plate $12 \times 180 \times 180$ mm made of the V95pchT2 alloy (plate 40 mm thick), which is subjected to pure twisting at $T = 180$°C under the action of a unit torque M equivalent to bending moments of different signs $M_1 = -M_2 = M$ uniformly distributed along the plate edges (Fig. 1). In the experiment, such a scheme may be achieved by applying the point forces $P = 2M$ at the plate corners, as is shown in Fig. 2 [3]. The filled and open points in Fig. 2 show the experimental values of the curvature χ as a function of time with ignored initial elastic component $\chi_0 = \chi(0)$ under the action of a unit torque $M = 4.843$ kN $\cdot$ m/m (data of two experiments with the same value of M). Figure 3 shows a plate being formed for two hours.

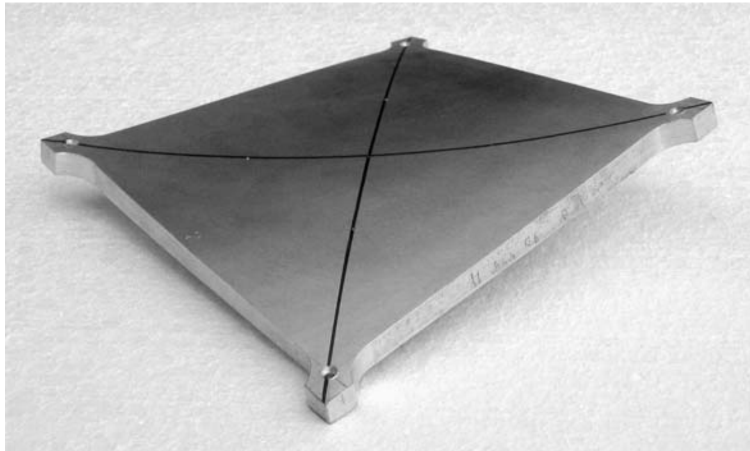

Fig. 3. Plate being formed for two hours.

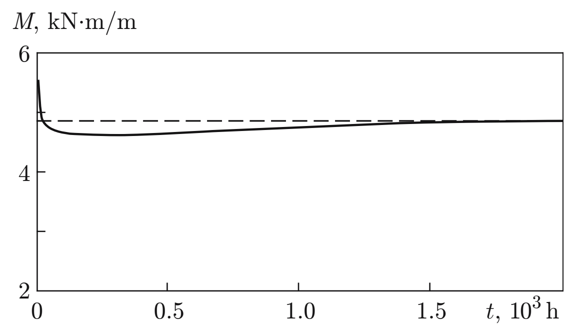

Fig. 4. Torque versus time: the dashed curve refers to the constant moment M and the solid curve refers to the calculation for the prescribed kinematic regime corresponding to the solid curve in Fig. 2.

To solve the problem of plate twisting, we used the in-house program CreePL developed by the authors of the present paper to calculate the shape formation of double-curvature panels in the kinematic formulation under conditions close to pure bending. The total strain ε_{kl} is the sum of elastic strains ε^e_{kl} and creep strains ε^c_{kl}:

$$\varepsilon^e_{kl} + \varepsilon^c_{kl} = -zw_{,kl}. \tag{5}$$

Here

$$\varepsilon^e_{kl} = \frac{1-\nu}{E}\,\sigma_{nn}\delta_{kl} + \frac{1+\nu}{E}\,\sigma^0_{kl}, \qquad \sigma^0_{kl} = \sigma_{kl} - \frac{1}{2}\,\sigma_{nn}\delta_{kl}, \qquad \dot{\varepsilon}^c_{kl} = \eta_{kl} \qquad (k,l = 1,2),$$

w is the deflection, and σ^0_{kl} and δ_{kl} are the components of the plane stress deviator and unit tensor; the subscript k after the comma indicates the derivative with respect to the coordinate x_k; summation is performed over the repeated indices from 1 to 2. The creep strain rates η_{kl} are determined in accordance with Eqs. (4). The axis Oz is perpendicular to the plane Ox_1x_2; $|z| \le h/2$ (h is the plate thickness).

The function $w = w(x_1, x_2, t)$ being known, relations (5) differentiated with respect to t, combined with Eqs. (4), form a system of ordinary differential equations with respect to time, which allow finding the stress components $\sigma_{kl} = \sigma_{kl}(x_1, x_2, z, t)$. The initial conditions $\sigma_{kl}|_{t=0}$ are found from Eq. (5) with $\varepsilon^c_{kl} = 0$ $(k,l = 1,2)$. This system is solved by the fourth-order Runge–Kutta–Merson method with the time step chosen automatically.

In the case of plate twisting considered, for the problem to be solved in the kinematic formulation, the deflection was set in the form $w = (A_1 + A_2 t/t_*)x_1x_2$, where -0.09 m $\le x_k \le 0.09$ m $(k = 1,2)$; $t_* = 2$ h.

The solid curve in Fig. 2 refer to the curvature corresponding to the regime with $A_1 = 0.95$ m^{-1} and $A_2 = 0.5$ m^{-1}. This dependence is plotted with ignored initial elastic component $\chi_0 = 0.8333$ m^{-1} corresponding to the experimental moment $M = 4.843$ kN $\cdot$ m/m. The values of A_1 and A_2 are chosen so that the torque (solid curve in Fig. 4) at $0 < t \le t_*$ is as close as possible to the value $M = 4.843$ kN $\cdot$ m/m (dashed curve in Fig. 4).

The dotted and dashed curves in Fig. 2 show similar calculations by the ANSYS program under the assumption that the material is isotropic and possesses identical properties under tension and compression; the dotted curve is the calculation based on material characteristics under pure tension, and the dashed curve is the calculation based on material characteristics under pure compression. These dependences yield the upper and lower estimates for the experiment and calculation performed with the use of Eqs. (4) and (5).

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